

# Internet Appendix for "Capital Share Risk in U.S. Asset Pricing"

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## I. Data Description

### CONSUMPTION

Consumption is measured as expenditures on nondurables and services, excluding shoes and clothing. The quarterly data are seasonally adjusted at annual rates, in billions of chain-weighted 2005 dollars. The components are chain-weighted together, and this series is scaled up so that the sample mean matches the sample mean of total personal consumption expenditures. Our source is the U.S. Department of Commerce, Bureau of Economic Analysis.

### LABOR SHARE

We use nonfarm business sector labor share throughout the paper. For nonfarm business sector, the methodology is summarized in Gomme and Rupert (2004). Labor share is measured as labor compensation divided by value added. The labor compensation is defined as Compensation of Employees - Government Wages and Salaries- Compensation of Employees of Nonprofit Institutions - Private Compensation (Households) - Farm Compensation of Employees - Housing Compensation of Employees - Imputed Labor Compensation of Self-Employed. The value added is defined as Compensation of Employees + Corporate Profits + Rental Income + Net Interest Income + Proprietors' Income + Indirect Taxes Less Subsidies + Depreciation. The quarterly, seasonally adjusted data spans from 1963:Q3 to 2013:Q4 with index 2009=100. The source is from Bureau of Labor Statistics. The labor share index is available at <http://research.stlouisfed.org/fred2/series/PRS85006173> and the quarterly LS level can be found from the dataset at [https://www.bls.gov/lpc/special\\_requests/msp\\_dataset.zip](https://www.bls.gov/lpc/special_requests/msp_dataset.zip).

### QUARTERLY RETURNS

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The return in quarter  $Q$  of year  $Y$ , denoted  $R_{Q,Y}$ , is the compounded monthly return over the three months in the quarter,  $m1, \dots, m3$ :

$$1 + R_{Q,Y} = \left(1 + \frac{R_{Q,Y}^{m1}}{100}\right) \left(1 + \frac{R_{Q,Y}^{m2}}{100}\right) \left(1 + \frac{R_{Q,Y}^{m3}}{100}\right)$$

As test portfolios, we use the excess return constructed by subtracting the quarterly 3-month Treasury bill rate from the above. The sample spans from 1963Q1 to 2013Q4.

#### FAMA FRENCH PRICING FACTORS

We obtain quarterly Fama French pricing factor HML, SMB, Rm, and risk free rates from professor French's online data library [http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/ftp/F-F\\_Benchmark\\_Factors\\_Quarterly.zip](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/ftp/F-F_Benchmark_Factors_Quarterly.zip). The sample spans 1963:Q3 to 2013:Q4.

#### LEVERAGE FACTOR

The broker-dealer leverage factor  $LevFac$  is constructed as follows. Broker-dealer ( $BD$ ) leverage is defined as

$$Leverage_t^{BD} = \frac{\text{Total Financial Assets}_t^{BD}}{\text{Total Financial Assets}_t^{BD} - \text{Total Liabilities}_t^{BD}}$$

The leverage factor is constructed as seasonally adjusted log changes

$$LevFac_t = [\Delta \log (Leverage_t^{BD})]^{SA}.$$

This variable is available from Tyler Muir's website over the sample used in Adrian, Etula, and Muir (2014), which is 1968:Q1-2009:Q4.<sup>1</sup> In this paper we use the larger sample 1963:Q3 to 2013:Q4. There are no negative observations on broker-dealer leverage in this sample. To extend the sample to 1963:Q3 to 2013:Q4 we use the original data on the total financial asset and liability of brokers and dealers data from flow of funds, Table L.128 available at <http://www.federalreserve.gov/apps/fof/DisplayTable.aspx?t=1.128>. Adrian, Etula, and Muir (2014) seasonally adjust  $\Delta \log (Leverage_t^{BD})$  by computing an expanding window regression of  $\Delta \log (Leverage_t^{BD})$  on dummies for three of the four quarters in the year at each date using the data up to that date. The initial series 1968Q1 uses data from previous 10 quarters in their sample and samples expand by recursively adding one observation on the end. Thus, the residual from this regression over the first subsample window 1965:Q3-1968:Q1 is taken as the observation for  $LevFac_{68:Q1}$ . An observation is added to the end

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<sup>1</sup>Link: [http://faculty.som.yale.edu/tylermuir/LEVERAGEFACTORDATA\\_001.txt](http://faculty.som.yale.edu/tylermuir/LEVERAGEFACTORDATA_001.txt)

and the process is repeated to obtain  $LevFac_{68:Q2}$ , and so on. We follow the same procedure (starting with the same initial window 1965:Q3-1968:Q1) to extend the sample forward to 2013Q4. To extend backwards to 1963:Q1, we take data on  $\Delta \log(Leverage_t^{BD})$  from 1963:Q1 to 1967:Q4 and regress on dummies for three of four quarters and take the residuals of this regression as the observations on  $LevFac_t$  for  $t = 1963:Q1-1967:Q4$ . Using this procedure, we exactly reproduce the series available on Tyler Muir's website for the overlapping subsample 1968:Q1 to 2009:Q4, with the exception of a few observations in the 1970s, a discrepancy we can't explain. To make the observations we use identical for the overlapping sample, we simply replace these few observations with the ones available on Tyler Muir's website.

#### HOUSEHOLD STOCK MARKET WEALTH

We obtain the stock market wealth data from two sources. The first is the triennial Survey of Consumer Finance (SCF) conducted by Board of Governors of the Federal Reserve System from 1989-2013. Stock Wealth includes both direct and indirect holdings of public stock. Stock wealth for each household is calculated according to the construction in SCF, which is the sum of following items: 1. directly-held stock. 2. stock mutual funds: full value if described as stock mutual fund, 1/2 value of combination mutual funds. 3. IRAs/Keoghs invested in stock: full value if mostly invested in stock, 1/2 value if split between stocks/bonds or stocks/money market, 1/3 value if split between. 4. other managed assets w/equity interest (annuities, trusts, MIAs): full value if mostly invested in stock, 1/2 value if split between stocks/MFs & bonds/CDs, or "mixed/diversified," 1/3 value if "other" stocks/bonds/money market. 5. thrift-type retirement accounts invested in stock full value if mostly invested in stock, 1/2 value if split between stocks and interest earning assets. 6. savings accounts classified as 529 or other accounts that may be invested in stocks. Households with a non-zero/non-missing stock wealth by any of the above are counted as a stockowner. All stock wealth values are in real terms adjusted to 2013 dollars. All summary statistics (mean, median, participation rate, etc.) are computed using SCF weights. In particular, in the original data, in order to minimize the measurement error, each household has five imputations. We follow the exact method suggested in SCF website by computing the desired statistic separately for each implicate using the sample weight (X42001). The final point estimate is given by the average of the estimates for the five implicates.

The second source is the Saez-Zucman (SZ) data on wealth inequality based on capitalized income tax data available at <http://gabriel-zucman.eu/uswealth/>. The SZ data

provides estimates of the distribution of wealth and income for all households but does not isolate the distributions for stockholders. To do so, we first download the replication package at <http://gabriel-zucman.eu/files/uswealth/SZreplic.zip> along with the yearly public-use micro-files available at the NBER at <http://users.nber.org/~taxsim/gdb/>. Following SZ, we supplement this dataset using the internal use Statistics of Income (SOI) individual tax return sample files from 1979 onward. We define stockholders to be individuals with non-zero dividends (*divinc*) and/or non-zero realized capital gain (*kginc*). Second, we follow the “mixed” method of capitalizing income from dividends and capital gains proposed by SZ. Specifically, when ranking households into wealth groups, only dividends (*divinc*) are capitalized. Thus, if in 2000 the ratio of equities to the sum of dividend income reported on tax returns is 54, then a family’s ranking in the wealth distribution is determined by taking its dividend income and multiplying by 54. By contrast, when computing the stock wealth of each percentile group, both dividends and capital gains are capitalized. Thus, if in 2000 the ratio of equities to the sum of dividend and capital gain income reported on tax returns is 10, a household’s equity wealth for that year is captured by multiplying its dividend and capital gains income by 10. The purpose of this mixed method given by SZ is to smooth realized capital gains and not overstate the concentration of wealth. We apply the linear interpolation for the data points in 1963 and 1965 that are missing in the NBER dataset.

#### HOUSEHOLD INCOME DATA

We obtain the household income data from two sources. The first is the SCF. We define total income as reported on the SCF is defined as the sum of three components.  $Y_t^i = Y_{i,t}^L + \widehat{Y_{i,t}^c + Y_{i,t}^o}$ . The mimicking factors for the income shares is computed by taking the fitted values  $\widehat{Y_t^i/Y_t}$  from regressions of  $Y_t^i/Y_t$  on  $(1 - LS_t)$  to obtain quarterly observations extending over the larger sample for which data on  $LS_t$  are available. We obtain the household income data from the triennial Survey of Consumer Finance (SCF) conducted by Board of Governors of the Federal Reserve System from 1989-2013. All the income is adjusted relative to 2013 dollars. Throughout the paper, we define the labor income as

$$Y_{i,t}^L \equiv wage_{i,t} + LS_t \times se_{i,t}$$

where  $wage_{i,t}$  is the labor wage at time  $t$  and  $se_{i,t}$  is the income from self-employment at time  $t$ , and  $LS_t$  is the labor share at time  $t$

Similarly, we define the capital income

$$Y_{i,t}^c \equiv se_{i,t} + int_{i,t} + div_{i,t} + cg_{i,t} + pension_{i,t}$$

where  $int_{i,t}$  is the taxable and tax-exempt interest,  $div$  is the dividends,  $cg$  is the realized capital gains and  $pension_{i,t}$  is the pensions and withdrawals from retirement accounts.

The other income is defined as

$$Y_{i,t}^o \equiv gov_{i,t} + ss_{i,t} + alm_{i,t} + others_{i,t}$$

where  $gov_{i,t}$  is the food stamps and other related support programs provided by government,  $ss_{i,t}$  is the social security,  $alm_{i,t}$  is the alimony and other support payments,  $others_{i,t}$  is the miscellaneous sources of income for all members of the primary economic unit in the household.

The second source is from Saez-Zucman at <http://gabriel-zucman.eu/uswealth/>. Similar to the wealth data, the SZ data provides estimates of the distribution of income for all households but does not isolate the distributions for stockholders. To do so, we first download the replication package at <http://gabriel-zucman.eu/files/uswealth/SZreplic.zip> along with yearly public-use micro-files available at the NBER at <http://users.nber.org/~taxsim/gdb/> and supplement this dataset using the internal use Statistics of Income (SOI) individual tax return sample files from 1979 onward. We calculate the total income of stockholders by isolating only those households with non-zero dividends ( $divinc$ ) and/or non-zero realized capital gain ( $kginc$ ). Total income is defined as the sum of capital income ( $Y_{i,t}^K$ ) and labor income ( $Y_{i,t}^L$ )

$$Y_{i,t} \equiv Y_{i,t}^K + Y_{i,t}^L$$

Capital income  $Y_{i,t}^K$  is defined as

$$Y_{i,t}^K \equiv div_{i,t} + int_{i,t} + rent_{i,t} + kbus_{i,t} + pen_{i,t}$$

where  $div_{i,t}$  is dividends ( $divkg\_na$ ),  $int_{i,t}$  is interest ( $int\_na$ ),  $rent_{i,t}$  is housing income ( $rent\_na$ ),  $kbus_{i,t}$  is the return on business wealth ( $kbus\_na$ ) and  $pen_{i,t}$  is pension income ( $pen\_na$ ). Labor income  $Y_{i,t}^L$  is defined as

$$Y_{i,t}^L \equiv wage_{i,t} + lbus_{i,t}$$

where  $wage_{i,t}$  is wage income ( $wag\_na$ ) and  $lbus_{i,t}$  is business income net of the return on business wealth.

We rank households into wealth groups by capitalized dividends ( $divinc$ ) as described above in the subsection “Household Stock Market Wealth” and calculate the total income

$Y_{i,t}$  for each group. We apply the linear interpolation for the data points in 1963 and 1965 that are missing in the NBER dataset.

## II. A Stylized Model of Asset Owners and Workers

We consider a stylized limited participation endowment economy in which wealth is concentrated in the hands of a few asset owners, or “shareholders,” while most households are “workers” who finance consumption out of wages and salaries. We consider a closed economy. Workers own no risky asset shares and consume their labor earnings. There is no risk-sharing between workers and shareholders. A representative firm issues no new shares and buys back no shares. Dividends are equal to output minus a wage bill:

$$D_t = Y_t - w_t N_t$$

where  $w_t$  equals the wage and  $N_t$  is aggregate labor supply. The wage bill is equal to  $Y_t$  times a time-varying labor share  $\alpha_t$ :

$$w_t N_t = \alpha_t Y_t \Rightarrow D_t = (1 - \alpha_t) Y_t. \quad (\text{A1})$$

We rule out short sales in the risky asset:

$$\theta_t^i \geq 0.$$

Asset owners not only purchase shares in the risky security, they also trade with one another in a one-period bond with price at time  $t$  denoted  $q_t$ . The real quantity of bonds are denoted  $B_{t+1}$ , where  $B_{t+1} < 0$  represents a borrowing position. The bond is in zero-net supply among asset owners. Asset owners could also have idiosyncratic investment income  $\zeta_t^i$ . The gross financial assets of investor  $i$  at time  $t$  is defined

$$A_t^i \equiv \theta_t^i (V_t + D_t) + B_t^i.$$

The budget constraint for the  $i$ th investor is

$$\begin{aligned} C_t^i + B_{t+1}^i q_t + \theta_{t+1}^i V_t &= A_t^i + \zeta_t^i \\ &= \theta_t^i (V_t + D_t) + B_t^i + \zeta_t^i, \end{aligned} \quad (\text{A2})$$

where  $C_t^i$  denotes the consumption of investor  $i$ .

A large number of identical non-rich workers, denoted by  $w$ , receive labor income do not participate in asset markets. The budget constraint for the representative worker is therefore

$$C^w = \alpha_t Y_t. \tag{A3}$$

Equity market clearing requires

$$\sum_i \theta_t^i = 1.$$

Bond market clearing requires

$$\sum_i B_t^i = 0.$$

Aggregating (A2) and (A3) and imposing market clearing and (A1) implies that aggregate (worker plus shareholder) consumption,  $C_t$ , is equal to total output  $Y_t$ . Aggregating over the budget constraint of the shareholders shows that their consumption is equal to the capital share times  $C_t$ :

$$C_t^S = D_t = \underbrace{(1 - \alpha_t)}_{KS_t} C_t.$$

A representative shareholder who owns the entire corporate sector will therefore have consumption equal to  $C_t \cdot KS_t$ . This reasoning goes through as an approximation if workers own a small fraction of the corporate sector even if there is some risk-sharing in the form of risk-free borrowing and lending between workers and shareholders, as long as any risk-sharing across these groups is imperfect. The point is that, while individual shareholders can smooth out transitory fluctuations in income by buying and selling assets, shareholders as a whole are less able to do so since purchases and sales of any asset must net to zero across all asset owners.

### III. Low Frequency Risk Exposures

This Section provides a parametric example of conditions under which longer horizon (e.g., multi-quarter) risk exposures more accurately measure the true short horizon (e.g., one-quarter) exposure in finite samples. We start with the SDF

$$M_t = \delta \left( \frac{C_t^s}{C_{t-1}^s} \right)^{-\gamma} \left( \frac{G_{t+1}}{G_t} \right)^{-\chi},$$

or

$$\log M_t = \log(\delta) - \gamma \Delta \ln C_t^s - \chi \Delta \ln G_t.$$

Using the approximation  $\log M_t \approx M_t - 1$ , we have

$$\begin{aligned} M_t &\approx 1 + \log(\delta) - \gamma \Delta \ln C_t^s - \chi \Delta \ln G_t \\ &\approx b_0 - \gamma \frac{C_t^s}{C_{t-1}^s} - \chi \frac{G_t}{G_{t-1}}, \end{aligned}$$

where  $b_0 = 1 + \log(\delta) - \gamma - \chi$ . This is an approximately linear two factor model with factors given by  $\frac{C_t^s}{C_{t-1}^s}$  and the latent  $\frac{G_t}{G_{t-1}}$ .

Let stockholder consumption be  $C_t^s = C_t K S_t$ , where  $C_t$  is aggregate (shareholder plus worker) consumption. Aggregate consumption growth is very stable compared to capital share growth in our sample. For the sake of illustration in this appendix, we assume it is constant. Then  $K S_t$  is the only source of variation in stockholder consumption growth and the two factors are now the latent  $\frac{G_t}{G_{t-1}}$  and  $\frac{K S_t}{K S_{t-1}}$ . We denote the true value of the parameters with superscript “o”. In this example, the data generating processes (DGPs) of gross returns  $R_{j,t+1}$ ,  $\frac{K S_t}{K S_{t-1}}$ , and  $\frac{G_t}{G_{t-1}}$  are presumed to follow

$$\begin{aligned} R_{j,t+1} &= 1 + \beta_G^o \frac{G_t}{G_{t-1}} + \beta_{KS,1}^o \frac{K S_t}{K S_{t-1}} + \zeta_{j,t+1} \\ \left( \frac{K S_{t+1}}{K S_t} - \mu_{KS}^o \right) &= \rho_{KS}^o \left( \frac{K S_t}{K S_{t-1}} - \mu_{KS}^o \right) + \varepsilon_{KS,t+1} \\ \left( \frac{G_{t+1}}{G_t} - \mu_G^o \right) &= \rho_G^o \left( \frac{G_t}{G_{t-1}} - \mu_G^o \right) + \varepsilon_{G,t+1} \end{aligned}$$

where  $\zeta_{j,t+1}$  is an idiosyncratic shock. The level of capital share growth appears extremely persistent in the data, with an estimated first order autoregressive root of 0.97, a series indistinguishable from one with a unit root in statistical tests. Since there are well known difficulties with simulating from a process with an autoregressive root that is local-to-unity, we instead simulate from a process calibrated to match autoregressive properties of the growth in the capital share, which is clearly stationary in the data, with a first order autoregressive coefficient of -0.25. It should be clear that an autoregressive coefficient of -0.25 in the first-differenced data is tautologically consistent with a data generating process that has an autoregressive root of 0.97 in levels.

We let  $\zeta_{j,t+1}$  be drawn from Normal distribution  $N(0, 1)$  and  $(\varepsilon_{KS,t+1}, \varepsilon_{G,t+1})$  be jointly



drawn from a bivariate Normal distribution, i.e.,

$$\begin{aligned}\zeta_{j,t} &\sim N(0, 1) \\ (\varepsilon_{G,t}, \varepsilon_{KS,t})' &\sim N(0, \Sigma)\end{aligned}$$

where

$$\Sigma = \begin{bmatrix} \sigma_G^2 & \sigma_{GKS} \\ \sigma_{GKS} & \sigma_{KS}^2 \end{bmatrix}$$

Because the latent factor  $\frac{G_{t+1}}{G_t}$  is omitted from the econometrician's set of risk factors, capital share risk exposures are estimated using the univariate regressions

$$R_{j,t+H,t} = a + \beta_{KS,H} \frac{KS_{t+H,t}}{KS_t} + u_{j,t+1},$$

for various  $H = 1, 2, \dots$ , where  $H$  represents the horizon over which returns and capital share growth are measured and  $R_{j,t+H,t}$  denotes the gross return from the end of  $t$  to the end of  $t + H$ . We now consider a parametric example intended to be illustrative of the conditions under which longer horizon risk exposures more accurately measure true risk exposures even at short horizons. The parametrization is given in the table below for two different values of the true one-period capital share exposure  $\beta_{KS,1}^0$ .

Parameters

$\beta_G^0$	$\beta_{KS,1}^0$	$\rho_{KS}$	$\rho_G$	$\sigma_G$	$\sigma_{KS}$	$\mu_G$	$\mu_{KS}$	$\sigma_{GKS}$
0.1	(0.10, -0.10)	-0.25	-0.5	1.5	0.45	1.10	1.05	-0.2

The calibration of  $\rho_{KS} = -0.25$  is set in order to match the estimated first order autocorrelation coefficient for capital share growth in the data. Consider a parameterization in which positive exposure to  $\frac{G_{t+1}}{G_t}$  earns a positive risk premium. In this case,  $\beta_G^0 > 0$ . The key aspects of the above parametrization are that  $\sigma_{GKS} < 0$  and  $\rho_G < \rho_{KS}$ . That is, the omitted factor is negatively correlated with the included factor  $\frac{KS_{t+H,t}}{KS_t}$  but more transitory than the included factor. The results for a sample size of  $T = 202$  as in our data are below. The estimated betas are reported as averages over  $N = 10,000$  samples for  $\hat{\beta}_{KS,H}$  for two values of  $\beta_{KS,1}^0$

Average Estimated  $\widehat{\beta}_{KS,H}$  across 10,000 Samples

$\beta_{KS,1}^0$	$H = 1$	$H = 4$	$H = 8$	$H = 12$	$H = 16$
0.10	0.040	0.055	0.080	0.096	0.097
-0.10	-0.0623	-0.071	-0.075	-0.078	-0.082

Under this parameter configuration,  $\widehat{\beta}_{KS,1}$  is biased down when the true exposure  $\beta_{KS,1}^0$  is positive and biased up when the true exposure is negative, thereby compressing spreads. But the long-horizon estimated exposures  $\widehat{\beta}_{KS,H}$  for  $H = 8$  or  $12$  are a better estimates of the true one-period exposure  $\beta_{KS,1}^0$ . The reason is that the long-horizon regressions attenuate the bias in short-horizon betas created by omitting the less persistent but more volatile  $G_{t+1}/G_t$ . This factor is a source of noise in the short-horizon regressions but is largely dissipated in the long-horizon relationships. Note that since the missing factor is by definition unknown and latent, the sign of any asset's risk exposure  $\beta_G^0$  is unidentified. We could just as well have presumed that the  $\beta_G^0 < 0$ , in which case the same example goes through with  $\sigma_{GKS}$  greater than zero rather than less than zero.

## IV. GMM Estimations

### A. Nonlinear SDF Estimation

Estimates of the benchmark nonlinear models are based on the following  $N + 1$  moment conditions

$$g_T(b) = \mathbb{E}_T \left[ \begin{array}{c} \mathbf{R}_t^e - \lambda_0 \mathbf{1}_N + \frac{(M_{t+H,t} - \mu_H) \mathbf{R}_{t+H,t}^e}{\mu_H} \\ M_{t+H,t} - \mu_H \end{array} \right] = \begin{bmatrix} \mathbf{0} \\ 0 \end{bmatrix} \quad (\text{A4})$$

where  $\mathbb{E}_T$  denotes the sample mean in a sample with  $T$  time series observations,  $\mathbf{R}_t^e = [R_{1,t}^e \dots R_{N,t}^e]'$  denotes an  $N \times 1$  vector of excess returns, and the parameters to be estimated are  $\mathbf{b} \equiv (\mu_H, \gamma, \lambda_0, \beta)'$ . The first  $N$  moments are the empirical counterparts to  $\mathbb{E}(R_{jt+1}^e) = \frac{-\text{Cov}(M_{t+1}, R_{t+1}^e)}{\mathbb{E}(M_{t+1})}$ , with two differences. First, the parameter  $\lambda_0$  (the same in each return equation) is included to account for a “zero beta” rate if there is no true risk-free rate and quarterly  $T$ -bills are not an accurate measure of the zero beta rate. Second, the equations to be estimated specify models in which *long*-horizon  $H$ -period empirical covariances between excess returns  $\mathbf{R}_{t+H,t}^e$  and the SDF  $M_{t+H,t}^k$  are used to explain *short*-horizon (quarterly)

average return premia  $\mathbb{E}(\mathbf{R}_t^e)$ . This implements the approach that is discussed in the text regarding low frequency risk exposures. We estimate models of the form (A4) for different values of  $H$ .<sup>2</sup>

The equations above are estimated using a weighting matrix consisting of an identity matrix for the first  $N$  moments, and a very large fixed weight on the last moment used to estimate  $\mu_H$ . By equally weighting the  $N$  Euler equation moments, we insure that the model is forced to explain spreads in the original test assets, and not spreads in re-weighted portfolios of these.<sup>3</sup> This is crucial for our analysis, since we seek to understand the large spreads on the specific portfolios of this study, not on re-weighted portfolios of these. However, it is important to estimate the mean of the stochastic discount factor accurately. Since the SDF is less volatile than stock returns, this requires placing a large (fixed) weight on the last moment.

For these estimations, we report a cross sectional  $R^2$  for the asset pricing block of moments as a measure of how well the model explains the cross-section of quarterly returns. This measure is defined as

$$R^2 = 1 - \frac{\text{Var}_c \left( \mathbb{E}_T (R_j^e) - \widehat{R}_j^e \right)}{\text{Var}_c \left( \mathbb{E}_T (R_i^e) \right)}$$

$$\widehat{R}_j^e = \widehat{\lambda}_0 + \frac{\mathbb{E}_T \left[ \left( \widehat{M}_{t+H,t}^k - \widehat{\mu}_H \right) R_{j,t+H,t}^e \right]}{\widehat{\mu}_H},$$

where  $\text{Var}_c$  denotes cross-sectional variance and  $\widehat{R}_j^e$  is the average return premium predicted by the model for asset  $j$ , and “hats” denote estimated parameters.

### B. Linear SDF Estimation

The nonlinear SDF is

$$M_{t+H,t} = \delta^H \left( \frac{C_{t+H}}{C_t} \right)^{-\gamma} \left( \frac{KS_{t+H}}{KS_t} \right)^{-\gamma}$$

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<sup>2</sup>This approach and underlying model are different than that taken by Parker and Julliard (2004), which studies covariances between short-horizon returns and *future* consumption growth over longer horizons. We don't pursue this approach here because such covariances are unlikely to capture low frequency components in the stock return-capital share relationship, which requires relating *long*-horizon returns to long-horizon SDFs.

<sup>3</sup>See Cochrane (2005) for a discussion of this issue.

We take a linear approximation of the above as follows. Taking logs, we have

$$\ln(M_{t+H,t}) = \ln(\delta^H) - \gamma \ln\left(\frac{C_{t+H}}{C_t}\right) - \gamma \ln\left(\frac{KS_{t+H}}{KS_t}\right).$$

Using  $\ln(1+x) \approx x$ , we have

$$\begin{aligned} M_{t+H,t} - 1 &\approx \ln(M_{t+H,t}) = \ln(\delta^H) - \gamma \ln\left(\frac{C_{t+H}}{C_t}\right) - \gamma \ln\left(\frac{KS_{t+H}}{KS_t}\right) \\ &\approx \ln(\delta^H) - \gamma \left(\frac{C_{t+H}}{C_t} - 1\right) - \gamma \left(\frac{KS_{t+H}}{KS_t} - 1\right) \end{aligned}$$

Or,

$$\begin{aligned} M_{t+H,t} &\approx \underbrace{[1 + \ln(\delta^H)]}_{b_0} - b_1 \left(\frac{C_{t+H}}{C_t} - 1\right) - b_2 \left(\frac{KS_{t+H}}{KS_t} - 1\right) \\ b_1 &= b_2 = \gamma. \end{aligned}$$

We use the above linearized  $M_{t+H,t}$  in GMM moment conditions (A4). However, since we are using excess return data,  $b_0$  and therefore the mean of the SDF  $\mu_H$  cannot be identified in the linear SDF specification. Thus we calibrate  $\delta = (0.95)^{\frac{1}{4}}$ , which pins down both  $b_0$  and  $\mu_H \equiv \mathbb{E}(M_{t+H,t}) = b_0 - b_1 \mathbb{E}\left(\frac{C_{t+H}}{C_t} - 1\right) - b_2 \mathbb{E}\left(\frac{KS_{t+H}}{KS_t} - 1\right)$ . We estimate three cases, (i)  $b_1 = b_2 = \gamma$  (ii)  $b_1 = 0, b_2 = \gamma$  (iii)  $b_1 = \gamma, b_2 = 0$  using the moment conditions

$$g_T(b) = \mathbb{E}_T \begin{bmatrix} \mathbf{R}_t^e - \lambda_0 \mathbf{1}_N + \frac{(M_{t+H,t} - \mu_H) \mathbf{R}_{t+H,t}^e}{\mathbb{E}(M_{t+H,t})} \\ \left(\frac{C_{t+H}}{C_t} - 1\right) - \mu_{c,H} \\ \left(\frac{KS_{t+H}}{KS_t} - 1\right) - \mu_{KS,H} \\ \left(\frac{C_{t+H}}{C_t} - 1\right) \left(\frac{KS_{t+H}}{KS_t} - 1\right) - \sigma_{C,KS} \\ \left(\frac{C_{t+H}}{C_t} - 1\right)^2 - \sigma_c^2 \\ \left(\frac{KS_{t+H}}{KS_t} - 1\right)^2 - \sigma_{KS}^2 \end{bmatrix} = \mathbf{0}.$$

The first block of moment conditions estimate the Euler equations, while the remaining blocks estimate the parameter elements of the covariance matrix of factors. The factor risk prices  $\lambda_H$  can be derived from

$$\begin{aligned}
\mathbb{E}(R_t^e) &= \lambda_0 - \frac{(M_{t+H,t} - \mu_H) \mathbf{R}_{t+H,t}^e}{\mu_H} \\
&= \lambda_0 + \frac{\text{Cov}(\mathbf{R}_{t+H,t}^e, f'_H) b}{\mu_H} \\
&= \lambda_0 + \frac{\text{Cov}(\mathbf{R}_{t+H,t}^e, f'_H) \text{Cov}(f_H, f'_H)^{-1} \text{Cov}(f_H, f'_H) b}{\mu_H} \\
&= \lambda_0 + \frac{\beta_H \text{Cov}(f_H, f'_H) b}{\mu_H},
\end{aligned}$$

where  $\mu_H = \mathbb{E}(M_{t+H,t})$ . It follows that

$$\lambda_H = \frac{\text{Cov}(f_H, f'_H) b}{\mu_H}.$$

The estimated  $\text{Cov}(f_H, f'_H)$  is

		$\text{Cov}(f'_H, f_H), f_H = \left( \frac{C_{t+H}}{C_{t-1}} - 1, \frac{KS_{t+H}}{KS_{t-1}} - 1 \right)$	
		all units are in multiple of 1000	
$H = 4$	0.1968	-0.0164	
	-0.0164	0.6709	
$H = 8$	0.5736	-0.1405	
	-0.1405	1.2184	

Table AII shows the cross-sectional explanatory power for quarterly expected returns of the model with the restriction  $b_1 = b_2$  imposed. Table AI shows that the estimates of  $\lambda_{C,H}$  are often several times smaller than those of  $\lambda_{KS,H}$  despite  $b_1 = b_2$ . From the estimates of  $\text{Cov}(f'_H, f_H)$ , we see the off-diagonal elements are small, implying that the correlation between the factors is low (equal to -0.04 for  $H = 4$  and -0.17 for  $H = 8$ ). With these estimates, an empirical model that eliminates the eliminates consumption growth from the SDF altogether is likely to perform about as well as one that includes it. Table AIII shows that this is what is found: little is lost in terms of cross-sectional  $R^2$  or pricing errors by estimating a model with  $b_1$  constrained to be zero, compared to the case where  $b_1 = b_2$  in Table AII. By contrast, dropping capital share growth from the SDF makes a big difference to the cross-section fit, as shown in AIV.

### C. Two Pass Regression GMM Estimation

Denote a generic vector of  $K$  factors for any model as  $\mathbf{f}_t$  (where  $K$  could be one, as in the capital share SDF). This appendix gives the general approach to our estimation of factor risk prices using two pass (time series and cross-sectional) regressions for any linear factor model.

The moment conditions for the expected return-beta representations are

$$g_T(\mathbf{b}) = \begin{bmatrix} \mathbb{E} \left( \underbrace{\mathbf{R}_{t+H,t}^e}_{N \times 1} - \underbrace{\mathbf{a}}_{N \times 1} - \underbrace{\boldsymbol{\beta}}_{(N \times K)} \underbrace{\mathbf{f}_t}_{(K \times 1)} \right) \\ \mathbb{E} \left( (\mathbf{R}_{t+H,t}^e - \mathbf{a} - \boldsymbol{\beta} \mathbf{f}_t) \otimes \mathbf{f}_t \right) \\ \mathbb{E} \left( \underbrace{\mathbf{R}_t^e}_{N \times 1} - \lambda_0 - \underbrace{\boldsymbol{\beta}}_{(N \times K)} \underbrace{\boldsymbol{\lambda}}_{(K \times 1)} \right) \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \quad (\text{A5})$$

where  $\mathbf{a} = [a_1 \dots a_N]'$  and  $\boldsymbol{\beta} = [\boldsymbol{\beta}_1 \dots \boldsymbol{\beta}_N]'$ , with parameter vector  $\mathbf{b}' = [\mathbf{a}, \boldsymbol{\beta}, \lambda_0, \boldsymbol{\lambda}]'$ . To obtain OLS time-series estimates of  $\mathbf{a}$  and  $\boldsymbol{\beta}$  and OLS cross sectional estimates of  $\lambda_0$  and  $\boldsymbol{\lambda}$ , we choose parameters  $\mathbf{b}$  to set the following linear combination of moments to zero

$$\mathbf{a}_T g_T(\mathbf{b}) = 0,$$

where

$$\mathbf{a}_T = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & [\mathbf{1}_N, \boldsymbol{\beta}]' \end{bmatrix}.$$

The point estimates from GMM are identical to those from Fama MacBeth regressions. To see this, in order to do OLS cross sectional regression of  $E(R_{i,t})$  on  $\boldsymbol{\beta}$ , recall that the first order necessary condition for minimizing the sum of squared residual is

$$\begin{aligned} \tilde{\boldsymbol{\beta}} \left( E(R_{i,t}) - \tilde{\boldsymbol{\beta}}[\lambda_0, \boldsymbol{\lambda}] \right) &= 0 \implies \\ [\lambda_0, \boldsymbol{\lambda}] &= \left( \tilde{\boldsymbol{\beta}}' \tilde{\boldsymbol{\beta}} \right)^{-1} \tilde{\boldsymbol{\beta}}' E(R_{i,t}) \end{aligned}$$

where  $\tilde{\boldsymbol{\beta}} = [\mathbf{1}_N, \boldsymbol{\beta}]$  to account for the intercept. If we multiply the first moment conditions with the identity matrix and the last moment condition with  $(K+1) \times N$  vector  $\tilde{\boldsymbol{\beta}}'$ , we will then have OLS time-series estimates of  $\mathbf{a}$  and  $\boldsymbol{\beta}$  and OLS cross sectional estimates of  $\lambda$ . To estimate the parameter vector  $\mathbf{b}$ , we set

$$\mathbf{a}_T g_T(\mathbf{b}) = 0$$

where

$$\underbrace{\mathbf{a}_T}_{\#Params \times \#Moments} = \begin{bmatrix} \underbrace{\mathbf{I}_{(K+1)N}}_{(K+1)N \times (K+1)N} & \underbrace{\mathbf{0}}_{(K+1)N \times N} \\ \underbrace{\mathbf{0}}_{(K+1) \times (K+1)N} & \underbrace{[\mathbf{1}_N, \boldsymbol{\beta}]'}_{(K+1) \times N} \end{bmatrix}$$

In order to use Hansen's formulas for standard errors, we compute the  $\mathbf{d}$  matrix of derivatives

$$\underbrace{\mathbf{d}}_{(K+2)N \times [(K+1)N+K+1]} = \frac{\partial g_T}{\partial \mathbf{b}'} = \begin{bmatrix} \underbrace{-\mathbf{I}_N}_{N \times N} & \underbrace{-\mathbf{I}_N \otimes \mathbb{E}(f_1) \quad \cdots \quad -\mathbf{I}_N \otimes \mathbb{E}(f_K)}_{N \times KN} & \underbrace{\mathbf{0}}_{N \times (K+1)} \\ -\mathbf{I}_N \otimes \mathbb{E}(f_1) & -\mathbf{I}_N \otimes \mathbb{E}(f_1^2) \quad \cdots \quad -\mathbf{I}_N \otimes \mathbb{E}(f_K f_1) & \underbrace{\mathbf{0}}_{KN \times (K+1)} \\ \vdots & \vdots \quad \ddots \quad \vdots & \underbrace{\mathbf{0}}_{KN \times (K+1)} \\ -\mathbf{I}_N \otimes \mathbb{E}(f_K) & -\mathbf{I}_N \otimes \mathbb{E}(f_1 f_K) \quad \cdots \quad -\mathbf{I}_N \otimes \mathbb{E}(f_K^2) & \underbrace{-[\mathbf{1}_N, \boldsymbol{\beta}]}_{N \times (K+1)} \\ \underbrace{\mathbf{0}}_{N \times N} & \underbrace{-\mathbf{I}_N \otimes \lambda'_1 \quad \cdots \quad -\mathbf{I}_N \otimes \lambda'_K}_{N \times KN} & \underbrace{-[\mathbf{1}_N, \boldsymbol{\beta}]}_{N \times (K+1)} \end{bmatrix}$$

We also need  $\mathbf{S}$  matrix, the spectral density matrix at frequency zero of the moment conditions

$$\mathbf{S} = \sum_{j=-\infty}^{\infty} E \left( \begin{bmatrix} \mathbf{R}_{t+H,t}^e - \mathbf{a} - \boldsymbol{\beta} \mathbf{f}_t \\ (\mathbf{R}_{t+H,t}^e - \mathbf{a} - \boldsymbol{\beta} \mathbf{f}_t) \otimes \mathbf{f}_t \\ \mathbf{R}_t^e - \lambda_0 - \boldsymbol{\beta} \boldsymbol{\lambda} \end{bmatrix} \begin{bmatrix} \mathbf{R}_{t+H-j,t-j}^e - \mathbf{a} - \boldsymbol{\beta} \mathbf{f}_{t-j} \\ (\mathbf{R}_{t+H-j,t-j}^e - \mathbf{a} - \boldsymbol{\beta} \mathbf{f}_{t-j}) \otimes \mathbf{f}_{t-j} \\ \mathbf{R}_{t-j}^e - \lambda_0 - \boldsymbol{\beta} \boldsymbol{\lambda} \end{bmatrix} \right).$$

Denote

$$h_t(\mathbf{b}) = \begin{bmatrix} \mathbf{R}_{t+H,t}^e - \mathbf{a} - \boldsymbol{\beta} \mathbf{f}_t \\ (\mathbf{R}_{t+H,t}^e - \mathbf{a} - \boldsymbol{\beta} \mathbf{f}_t) \otimes \mathbf{f}_t \\ \mathbf{R}_t^e - \lambda_0 - \boldsymbol{\beta} \boldsymbol{\lambda} \end{bmatrix}.$$

We employ a Newey west correction to the standard errors with lag  $L$  by using the estimate

$$\mathbf{S}_T = \sum_{j=-L}^L \left( \frac{L-|j|}{L} \right) \frac{1}{T} \sum_{t=1}^T h_t(\hat{\mathbf{b}}) h_{t-j}(\hat{\mathbf{b}})'$$

Asymptotic standard errors for the factor risk price estimates,  $\lambda$ , can be obtained using Hansen's formula for the sampling distribution of the parameter estimates

$$\underbrace{Var(\hat{\mathbf{b}})}_{[(K+1)N+K+1] \times [(K+1)N+K+1]} = \frac{1}{T} (\mathbf{a}_T \mathbf{d})^{-1} \mathbf{a}_T \mathbf{S}_T \mathbf{a}_T' (\mathbf{a}_T \mathbf{d})^{-1}.$$

## IV. Bootstrap Procedure

This section describes the bootstrap procedure for assessing the small sample distribution of cross-sectional  $R^2$  statistics. The bootstrap consists of the following steps.

1. For each test asset  $j$ , we estimate the time-series regressions on historical data for each  $H$  period exposure we study:

$$R_{j,t+H,t}^e = a_{j,H} + \beta_{j,KS,H} ([KS_{t+H}] / [KS_t]) + u_{j,t+H,t}. \quad (\text{A6})$$

We obtain the full-sample estimates of the parameters of  $a_{j,H}$  and  $\beta_{j,KS,H}$ , which we denote  $\widehat{a}_{j,H}$  and  $\widehat{\beta}_{j,KS,H}$ .

2. We estimate an AR(1) model for capital share growth also on historical data:

$$\frac{KS_{t+H}}{KS_t} = a_{KG,H} + \rho_H \left( \frac{KS_{t+H-1}}{KS_{t-1}} \right) + e_{t+H,t}. \quad (\text{A7})$$

3. We estimate  $\lambda_0$  and  $\lambda$  using historical data from cross-sectional regressions

$$E(R_{j,t}^e) = \lambda_0 + \lambda \widehat{\beta}_{j,KS,H} + \epsilon_j$$

where  $R_{j,t}^e$  is the quarterly excess return. From this regression we obtain the cross sectional fitted errors  $\{\widehat{\epsilon}_j\}_j$  and historical sample estimates  $\widehat{\lambda}_0$  and  $\widehat{\lambda}$ .

4. For each test asset  $j$ , we draw randomly with replacement from blocks of the fitted residuals from the above time-series regressions:

$$\begin{bmatrix} \widehat{u}_{1,1+H,1} & \cdots & \widehat{u}_{N,1+H,1} & \widehat{e}_{1+H,1} \\ \widehat{u}_{1,2+H,2} & & \widehat{u}_{N,2+H,2} & \widehat{e}_{2+H,2} \\ \vdots & & \vdots & \vdots \\ \widehat{u}_{1,T,T-H} & \cdots & \widehat{u}_{N,T,T-H} & \widehat{e}_{T,T-H} \end{bmatrix} \quad (\text{A8})$$

The  $m$ th bootstrap sample  $\left\{ u_{1,t+H,t}^{(m)}, \dots, \widehat{u}_{N,T,T-H}^{(m)}, e_{t+H,t}^{(m)} \right\}_{t=1}^H$  is obtained by sampling blocks of the raw data randomly with replacement and laying them end-to-end in the order sampled until a new sample of observations of length equal to the historical dataset is obtained. To choose the block length, we follow the recommendation of Hall, Horowitz, and Jing (1995) who show that the asymptotically optimal block length for estimating a symmetrical distribution function is  $l \propto T^{1/5}$ ; also see Horowitz (2003). For the results reported in the



text, we use a block length exactly equal to  $T^{1/5} = 3$ , but we check the robustness of our results to lengths of 5, 8, 12, and 20 and find little difference in the resulting confidence sets.

Next we recursively generate new data series for  $\frac{KS_{t+H}}{KS_t}$  by combining the initial value of  $\frac{KS_{1+H}}{K_1}$  in our sample along with the estimates from historical data  $\hat{a}_{KG,H}$ ,  $\hat{\rho}_H$  and the new sequence of errors  $\left\{e_{t+H,t}^{(m)}\right\}_t$  thereby generating an  $m$ th bootstrap sample on capital share growth  $\left\{\left(\frac{KS_{t+H}}{KS_t}\right)^{(m)}\right\}_t$ . We then generate new samples of observations on long-horizon returns  $\left\{R_{j,t+H,t}^{(m)}\right\}_t$  from new data on  $\left\{u_{j,t+H,t}^{(m)}\right\}_t$  and  $\left\{\left(\frac{KS_{t+H}}{KS_t}\right)^{(m)}\right\}_t$  and the sample estimates  $\hat{a}_{j,H}$  and  $\hat{\beta}_{j,KS,H}$ .

5. We generate  $m$ th observation  $\beta_{j,KS,H}^{(m)}$  from regression of  $\left\{R_{j,t+H,t}^{e(m)}\right\}_t$  on  $\left\{\left(\frac{KS_{t+H}}{KS_t}\right)^{(m)}\right\}_t$  and a constant.

6. We obtain an  $m$ th bootstrap sample  $\left\{\epsilon_j^{(m)}\right\}_j$  by sampling the fitted errors  $\{\hat{\epsilon}_j\}_j$  randomly with replacement and laying them end-to-end in the order sampled until a new sample of observations of length  $N$  equal to the historical cross-sectional sample is obtained. We then generate new samples of observations on quarterly average excess returns  $\left\{E\left(R_{j,t}^{e(m)}\right)\right\}_j$  from new data on  $\left\{\epsilon_j^{(m)}\right\}_j$  and  $\left\{\beta_{j,KS,H}^{(m)}\right\}_j$  and the sample estimates  $\hat{\lambda}_0$  and  $\hat{\lambda}$ .

7. We form the  $m$ th estimates  $\lambda_0^{(m)}$  and  $\lambda^{(m)}$  by regressing  $\left\{E\left(R_{j,t}^{e(m)}\right)\right\}_j$  on the  $m$ th observation  $\left\{\beta_{j,KS,H}^{(m)}\right\}_j$  and a constant. We store the  $m$ th sample cross-sectional  $\bar{R}^2$ ,  $\bar{R}^{(m)2}$  along with the  $m$ th values of  $\lambda_0^{(m)}$  and  $\lambda^{(m)}$ .

8. We repeat steps 4-7 10,000 times, and report the 95% confidence intervals for  $\left\{\bar{R}^{(m)2}, \lambda_0^{(m)}, \lambda^{(m)}\right\}_m$ .

#### A. Procedure Controlling for Other Pricing Factors

The bootstrap for cross-sectional regressions in which we control for other pricing factors is modified as follows.

1. Follow steps 1-5 separately for  $KS$  and the additional pricing factor(s)  $f$  and generate  $\beta_{j,KS,H}^{(m)}$  and  $\beta_{j,f,H}^{(m)}$  for the  $m$ th bootstrap.

2. Obtain an  $m$ th bootstrap sample  $\left\{\epsilon_j^{(m)}\right\}_j$  from the cross-sectional regression

$$E\left(R_{j,t}^e\right) = \lambda_0 + \lambda_{KS}\hat{\beta}_{j,KS,H} + \lambda_{HS}\beta_{j,f,H} + \epsilon_j.$$

As before, sample the fitted errors  $\{\hat{\epsilon}_j\}_j$  randomly with replacement, laying them end-to-end in the order sampled until a new sample of observations of length  $N$  equal to the historical

cross-sectional sample is obtained. Generate new samples of observations on quarterly average excess returns  $\left\{ E \left( R_{j,t}^{e(m)} \right) \right\}_j$  from new data on  $\left\{ \epsilon_j^{(m)} \right\}_j$  and  $\left\{ \beta_{j,KS,H}^{(m)}, \beta_{j,f,H}^{(m)} \right\}_j$  and the sample estimates  $\hat{\lambda}_0$ ,  $\hat{\lambda}_{KS}$  and  $\lambda_{HS}$

3. Form the  $m$ th estimates  $\lambda_0^{(m)}$  and  $\lambda^{(m)} = \left( \lambda_{KS}^{(m)}, \lambda_f^{(m)} \right)$  by regressing  $\left\{ E \left( R_{j,t}^{e(m)} \right) \right\}_j$  on the  $m$ th observation  $\left\{ \beta_{j,KS,H}^{(m)}, \beta_{j,f,H}^{(m)} \right\}_j$  and a constant. We store the  $m$ th sample cross-sectional  $\bar{R}^2$ ,  $\bar{R}^{(m)2}$ .

4. We repeat steps 1-3 10,000 times, and report the 95% confidence interval of  $\left\{ \bar{R}^{(m)2}, \lambda_{KS}^{(m)}, \lambda_f^{(m)} \right\}_m$ .

#### *B. Bootstrap Under the Null of No Cross-Sectional Explanatory Power*

We also conducted a bootstrap simulation under the null hypothesis that  $\beta_{j,KS,1} = \bar{\beta}_{KS,1}$  for all  $j$ . The steps in the bootstrap are the same as above with the following exceptions: in Step 1 we estimate the time-series regressions on historical data for  $H = 1$  period exposures and calibrate  $\beta_{j,KS,1}$  to be the average value across assets, for all  $j$ . In Step 3, we set  $\lambda_{KS} = 0$ , so the portfolios are completely independent. One period returns are then cumulated up to  $H$  period returns and the bootstrap confidence intervals under the null of no cross-sectional explanatory power computed. Table AVII reported below shows that the 95% bootstrapped confidence interval for the cross-sectional  $\bar{R}^2$  under the no explanatory power null ranges from values close to zero to values typically around 0.17 or smaller. By contrast, the estimated  $\bar{R}$  are much higher and fall well outside these ranges. The REV portfolios exhibit the largest ranges for the cross-sectional  $\bar{R}$  under the null with the upper end of the range about 0.4. These values are still much smaller than the estimated  $\bar{R}$  for these portfolios. In short, the magnitude of explanatory power we find is too large to be accounted for by sampling error in samples of the size we currently have.

## V. Appendix Tables and Figures

**Table AI: GMM Estimation of Linear Capital Share SDF**

This table reports estimates of risk prices  $\lambda_H$ . All estimates are multiplied by 100. The estimated  $b$  is from GMM estimation imposing  $b_1 = b_2$ . Serial correlation and heteroskedasticity robust  $t$ -statistics are reported in parenthesis. \*\* and \* indicate significance at 5% and 10% or better level, respectively. The sample spans the period 1963Q3 to 2013Q4.

$$\text{GMM, Linear SDF with } f'_H = \left( \frac{C_{t+H}}{C_t}, \frac{KS_{t+H}}{KS_t} \right)$$


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$$\lambda_H = -\mathbb{E}(M_{t+H,t})^{-1} \text{Cov}(f'_H, f_H) b, b = [b_1, b_2]', b_1 = b_2$$


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	Panel A: <b>Size/BM</b>		Panel B: <b>REV</b>		Panel C: <b>Size/INV</b>	
$H$	4	8	4	8	4	8
$\lambda_{C,H}$	0.17** (2.22)	0.15** (2.81)	0.15* (1.82)	0.12* (1.69)	0.14* (1.68)	0.18* (1.77)
$\lambda_{KS,H}$	0.61** (2.35)	0.53** (2.97)	0.53 (1.79)	0.30 (1.59)	0.49* (1.76)	0.44* (1.92)
	Panel D: <b>Size/OP</b>		Panel E: <b>All Equities</b>		Panel F: <b>Bonds</b>	
$H$	4	8	4	8	4	8
$\lambda_{C,H}$	0.16* (1.72)	0.18 (1.36)	0.15* (1.93)	0.17** (2.01)	0.13* (1.95)	0.11* (1.72)
$\lambda_{KS,H}$	0.57* (1.84)	0.45 (1.50)	0.55** (2.02)	0.43** (2.19)	0.56* (1.74)	0.31 (1.52)
	Panel G: <b>Sovereign Bonds</b>		Panel H: <b>Options</b>		Panel I: <b>CDS</b>	
$H$	4	8	4	8	4	8
$\lambda_{C,H}$	0.04 (0.34)	0.07 (0.81)	0.11 (1.03)	1.17 (1.31)	0.19 (1.46)	0.34* (1.74)
$\lambda_{KS,H}$	0.92 (1.18)	0.52 (1.07)	1.01** (2.25)	0.71* (1.81)	0.78 (1.24)	0.59* (1.75)

**Table AII: GMM Estimation of Linear Capital Share SDF**

This table reports GMM estimates of linear KS SDF. The cross sectional  $R^2$  is defined as  $R^2 = 1 - \frac{Var_c(\mathbb{E}(R_i^e) - \widehat{R}_i^e)}{Var_c(\mathbb{E}(R_i^e))}$ , where the fitted value  $\widehat{R}_i^e = \widehat{\alpha} + \frac{\mathbb{E}[(M_{t+H,t}^k - \widehat{\mu})\mathbf{R}_{t+H,t}^e]}{\widehat{\mu}}$ . The pricing error is defined as  $RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (\mathbb{E}(R_i^e) - \widehat{R}_i^e)^2}$  and  $RMSR = \sqrt{\frac{1}{N} \sum_{i=1}^N (\mathbb{E}(R_i^e))^2}$ . \*\* and \* indicate significance at 5% and 10% or better level, respectively. Serial correlation and heteroskedasticity robust  $t$ -statistics are reported in parenthesis. The sample spans the period 1963Q3 to 2013Q4.

<b>GMM, Linear SDF with <math>f'_H = \left( \frac{C_{t+H}}{C_t}, \frac{KS_{t+H}}{KS_t} \right)</math></b>						
SDF: $M_{t+H,t} = b_0 - b_1 \left( \frac{C_{t+H}}{C_t} - 1 \right) - b_2 \left( \frac{KS_{t+H}}{KS_t} - 1 \right)$						
$b_1 = b_2 = b$						
	Panel A: <b>Size/BM</b>		Panel B: <b>REV</b>		Panel C: <b>Size/INV</b>	
$H$	4	8	4	8	4	8
$b$	7.38**	3.21**	6.57**	2.24*	6.16**	3.14**
	(2.69)	(3.84)	(2.09)	(1.83)	(1.97)	(2.42)
$R^2$	0.56	0.83	0.64	0.83	0.41	0.69
$\frac{RMSE}{RMSR}$	0.20	0.12	0.14	0.09	0.21	0.15
	Panel D: <b>Size/OP</b>		Panel E: <b>All Equities</b>		Panel F: <b>Bonds</b>	
$H$	4	8	4	8	4	8
$b$	6.95**	3.17*	6.74**	3.04**	7.82**	2.52
	(2.09)	(1.91)	(2.29)	(2.74)	(2.06)	(1.64)
$R^2$	0.59	0.62	0.53	0.73	0.76	0.69
$\frac{RMSE}{RMSR}$	0.17	0.17	0.19	0.15	0.23	0.26
	Panel G: <b>Sovereign Bonds</b>		Panel H: <b>Options</b>		Panel I: <b>CDS</b>	
$H$	4	8	4	8	4	8
$b$	13.37*	4.11	15.90**	5.99**	12.22*	5.32**
	(1.80)	(1.43)	(3.84)	(2.99)	(1.74)	(2.14)
$R^2$	0.85	0.84	0.97	0.96	0.33	0.52
$\frac{RMSE}{RMSR}$	0.18	0.17	0.14	0.15	0.75	0.63

**Table AIII: GMM estimation of Linear Capital Share SDF**

This table reports GMM estimates of linear KS SDF. The cross sectional  $R^2$  is defined as  $R^2 = 1 - \frac{Var_c(\mathbb{E}(R_i^e) - \widehat{R}_i^e)}{Var_c(\mathbb{E}(R_i^e))}$ , where the fitted value  $\widehat{R}_i^e = \widehat{\alpha} + \frac{\mathbb{E}[(M_{t+H,t}^k - \widehat{\mu})\mathbf{R}_{t+H,t}^e]}{\widehat{\mu}}$ . The pricing error is defined as  $RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (\mathbb{E}(R_i^e) - \widehat{R}_i^e)^2}$  and  $RMSR = \sqrt{\frac{1}{N} \sum_{i=1}^N (\mathbb{E}(R_i^e))^2}$ . \*\* and \* indicate significance at 5% and 10% level or better level, respectively. Serial correlation and heteroskedasticity robust  $t$ -statistics are reported in parenthesis. The sample spans the period 1963Q3 to 2013Q4.

$$\text{GMM, Linear SDF with } f'_H = \begin{pmatrix} \frac{KS_{t+H}}{KS_t} \end{pmatrix}$$

SDF: $M_{t+H,t} = b_0 - b_1 \left( \frac{C_{t+H}}{C_t} - 1 \right) - b_2 \left( \frac{KS_{t+H}}{KS_t} - 1 \right)$						
$b_1 = 0$						
	Panel A: <b>Size/BM</b>		Panel B: <b>REV</b>		Panel C: <b>Size/INV</b>	
$H$	4	8	4	8	4	8
$b$	10.10**	4.90**	8.48*	2.65	8.15	3.94*
	(1.99)	(2.96)	(1.82)	(1.59)	(1.62)	(1.86)
$R^2$	0.51	0.81	0.74	0.88	0.40	0.62
$\frac{RMSE}{RMSR}$	0.21	0.13	0.12	0.08	0.21	0.17
	Panel D: <b>Size/OP</b>		Panel E: <b>All Equities</b>		Panel F: <b>Bonds</b>	
$H$	4	8	4	8	4	8
$b$	9.47*	4.17	9.15*	4.12**	12.32*	4.03*
	(1.89)	(1.53)	(1.89)	(2.05)	(1.81)	(1.86)
$R^2$	0.77	0.77	0.56	0.73	0.88	0.86
$\frac{RMSE}{RMSR}$	0.13	0.13	0.19	0.15	0.17	0.17
	Panel G: <b>Sovereign Bonds</b>		Panel H: <b>Options</b>		Panel I: <b>CDS</b>	
$H$	4	8	4	8	4	8
$b$	19.41	5.59*	29.16**	12.04**	18.62*	7.15**
	(1.46)	(1.78)	(2.74)	(2.11)	(1.92)	(2.53)
$R^2$	0.86	0.58	0.95	0.81	0.82	0.94
$\frac{RMSE}{RMSR}$	0.17	0.27	0.18	0.35	0.38	0.23

**Table AIV: GMM estimation of Linear Capital Share SDF**

This table reports GMM estimates of linear KS SDF. The cross sectional  $R^2$  is defined as  $R^2 = 1 - \frac{Var_c(\mathbb{E}(R_i^e) - \widehat{R}_i^e)}{Var_c(\mathbb{E}(R_i^e))}$ , where the fitted value  $\widehat{R}_i^e = \widehat{\alpha} + \frac{\mathbb{E}[(M_{t+H,t}^k - \widehat{\mu})\mathbf{R}_{t+H,t}^e]}{\widehat{\mu}}$ . The pricing error is defined as  $RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (\mathbb{E}(R_i^e) - \widehat{R}_i^e)^2}$  and  $RMSR = \sqrt{\frac{1}{N} \sum_{i=1}^N (\mathbb{E}(R_i^e))^2}$ . \*\* and \* indicate significance at 5% and 10% or better level, respectively. Serial correlation and heteroskedasticity robust  $t$ -statistics are reported in parenthesis. The sample spans the period 1963Q3 to 2013Q4.

**GMM, Linear SDF with  $f'_H = \begin{pmatrix} C_{t+H} \\ C_t \end{pmatrix}$**

SDF: $M_{t+H,t} = b_0 - b_1 \left( \frac{C_{t+H}}{C_t} - 1 \right) - b_2 \left( \frac{KS_{t+H}}{KS_t} - 1 \right)$						
$b_2 = 0$						
	Panel A: <b>Size/BM</b>		Panel B: <b>REV</b>		Panel C: <b>Size/INV</b>	
$H$	4	8	4	8	4	8
$b$	15.11**	4.53**	-4.70	2.19	10.46*	2.93
	(2.66)	(2.21)	(-0.35)	(0.88)	(1.92)	(1.47)
$R^2$	0.30	0.33	0.00	0.01	0.13	0.11
$\frac{RMSE}{RMSR}$	0.25	0.25	0.23	0.22	0.25	0.26
	Panel D: <b>Size/OP</b>		Panel E: <b>All Equities</b>		Panel F: <b>Bonds</b>	
$H$	4	8	4	8	4	8
$b$	-8.87	-1.41	7.95	2.69*	10.52	2.09
	(-0.66)	(-0.49)	(1.64)	(1.69)	(1.25)	(0.92)
$R^2$	0.06	0.02	0.07	0.10	0.17	0.07
$\frac{RMSE}{RMSR}$	0.26	0.28	0.27	0.27	0.43	0.45
	Panel G: <b>Sovereign Bonds</b>		Panel H: <b>Options</b>		Panel I: <b>CDS</b>	
$H$	4	8	4	8	4	8
$b$	7.04	2.69	34.40**	10.73*	-47.05	-10.38
	(0.69)	(0.78)	(2.48)	(1.91)	(-0.89)	(-1.48)
$R^2$	0.05	0.20	0.99	0.99	0.45	0.28
$\frac{RMSE}{RMSR}$	0.44	0.37	0.09	0.08	0.68	0.76

**Table AV: GMM Estimation of Capital Share SDF**

This table reports estimates of risk prices  $\lambda_H$ . All estimates are multiplied by 100. The estimated  $\mathbf{b}$  is from GMM estimation imposing  $b_1 = 0$ . Serial correlation and heteroskedasticity robust  $t$ -statistics are reported in parenthesis. \*\* and \* indicate significance at 5% and 10% or better level, respectively. The sample spans the period 1963Q3 to 2013Q4.

**GMM, Capital Share SDF**

$\lambda_H = -\mathbb{E}(M_{t+H,t})^{-1} \mathbf{Cov}(\mathbf{f}_H, \mathbf{f}'_H) \mathbf{b}, \mathbf{b} = [b_1, b_2]', b_1 = 0$						
	Panel A: <b>Size/BM</b>		Panel B: <b>REV</b>		Panel C: <b>Size/INV</b>	
$H$	4	8	4	8	4	8
$\lambda_{KS,H}$	0.74**	0.69**	0.62*	0.37	0.59	0.55*
	(2.00)	(2.82)	(1.77)	(1.52)	(1.61)	(1.82)
	Panel D: <b>Size/OP</b>		Panel E: <b>All Equities</b>		Panel F: <b>Bonds</b>	
$H$	4	8	4	8	4	8
$\lambda_{KS,H}$	0.69*	0.58	0.67*	0.57**	0.81*	0.54*
	(1.90)	(1.51)	(1.90)	(2.00)	(1.87)	(1.95)
	Panel G: <b>Sovereign Bonds</b>		Panel H: <b>Options</b>		Panel I: <b>CDS</b>	
$H$	4	8	4	8	4	8
$\lambda_{KS,H}$	1.50	0.99*	1.87**	1.72*	1.24*	0.83**
	(1.36)	(1.95)	(2.41)	(1.66)	(1.81)	(2.93)

**Table AVI: Nonlinear GMM Estimation of Capital Share SDF**

This table reports estimates of risk prices  $\lambda_H$ . The cross sectional  $R^2$  is defined as  $R^2 = 1 - \frac{Var_c(\mathbb{E}(R_i^e) - \widehat{R}_i^e)}{Var_c(\mathbb{E}(R_i^e))}$ , where the fitted value  $\widehat{R}_i^e = \widehat{\alpha} + \frac{\mathbb{E}[(M_{t+H,t}^k - \widehat{\mu})\mathbf{R}_{t+H,t}^e]}{\widehat{\mu}}$ . The pricing error is defined as  $RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (\mathbb{E}(R_i^e) - \widehat{R}_i^e)^2}$  and  $RMSR = \sqrt{\frac{1}{N} \sum_{i=1}^N (\mathbb{E}(R_i^e))^2}$ . \*\* and \* indicate significance at 5% and 10% or better level, respectively.  $\lambda_0$  is reported in unit of 100. Serial correlation and heteroskedasticity robust  $t$ -statistics are reported in parenthesis. The sample spans the period 1963Q3 to 2013Q4.

$$\text{GMM, Nonlinear SDF with } f'_H = \left( \frac{C_{t+H}}{C_t}, \frac{KS_{t+H}}{KS_t} \right)$$

SDF: $M_{t+H,t} = \delta^H \left( \frac{C_{t+H}}{C_t} \frac{KS_{t+H}}{KS_t} \right)^{-\gamma}$						
	Panel A: <b>Size/BM</b>		Panel B: <b>REV</b>		Panel C: <b>Size/INV</b>	
	4	8	4	8	4	8
$H$	4	8	4	8	4	8
$\lambda_0$	-0.07	0.66	0.42	1.14	0.42	0.67
	(-0.07)	(0.64)	(0.36)	(1.30)	(0.39)	(0.70)
$\gamma$	10.41**	4.46**	8.14	2.93	8.13	4.54**
	(2.19)	(3.27)	(1.54)	(1.59)	(1.54)	(2.18)
$R^2$	0.56	0.84	0.57	0.84	0.40	0.71
$\frac{RMSE}{RMSR}$	0.20	0.12	0.15	0.09	0.21	0.15
	Panel D: <b>Size/OP</b>		Panel E: <b>All Equities</b>		Panel F: <b>Bonds</b>	
	4	8	4	8	4	8
$\lambda_0$	-0.13	0.63	0.14	0.75	0.38	0.25
	(-0.12)	(0.63)	(0.14)	(0.82)	(1.63)	(1.20)
$\gamma$	10.16*	4.48	9.28*	4.23**	9.31*	3.10
	(1.73)	(1.49)	(1.85)	(2.46)	(1.75)	(1.48)
$R^2$	0.63	0.62	0.53	0.74	0.76	0.68
$\frac{RMSE}{RMSR}$	0.16	0.17	0.19	0.15	0.23	0.26
	Panel G: <b>Sovereign Bonds</b>		Panel H: <b>Options</b>		Panel I: <b>CDS</b>	
	4	8	4	8	4	8
$H$	4	8	4	8	4	8
$\lambda_0$	0.20	0.41	-1.56	-0.29	-0.18**	-0.30**
	(0.27)	(0.75)	(-1.46)	(-0.23)	(-2.48)	(-3.70)
$\gamma$	16.41	5.45	23.70**	9.02**	14.34	7.44
	(1.49)	(1.18)	(2.30)	(2.15)	(1.27)	(1.59)
$R^2$	0.88	0.83	0.96	0.96	0.30	0.49
$\frac{RMSE}{RMSR}$	0.16	0.17	0.17	0.16	0.76	0.64



**Table AVII: Bootstrap under the Null**

This table reports estimates of risk prices  $\lambda_H$ . All estimates are multiplied by 100. Bootstrap 95% confidence intervals are reported in square brackets under the null of no cross-sectional explanatory power. The sample spans the period 1963Q3 to 2013Q4.

**Expected Return-Beta Regressions, Bootstrap under the Null**

$\mathbb{E}(R_{j,t}^e) = \lambda_0 + \lambda_H' \beta_H + \epsilon_j$ , Estimates of Factor Risk Prices $\lambda_H$								
<b>Panel A: Size/BM</b>					<b>Panel B: REV</b>			
$H$	Constant	$\frac{KS_{t+H}}{KS_t}$	$\bar{R}^2$	$\frac{RMSE}{RMSR}$	Constant	$\frac{KS_{t+H}}{KS_t}$	$\bar{R}^2$	$\frac{RMSE}{RMSR}$
4	0.65	0.74	0.51	0.19	0.83	0.63	0.70	0.11
Base	[0.01, 1.23]	[0.42, 1.08]	[0.13, 0.77]		[0.35, 1.32]	[0.33, 0.92]	[0.17, 0.91]	
Under Null	[-0.11, 1.40]	[-0.01, 0.01]	[-0.04, 0.16]		[0.16, 1.50]	[-0.02, 0.02]	[-0.12, 0.43]	
8	1.55	0.68	0.80	0.12	1.73	0.41	0.86	0.08
Base	[1.39, 1.71]	[0.53, 0.83]	[0.52, 0.91]		[1.62, 1.84]	[0.30, 0.50]	[0.68, 0.96]	
Under Null	[1.39, 1.72]	[-0.00, 0.00]	[-0.04, 0.16]		[1.59, 1.86]	[-0.05, 0.04]	[-0.12, 0.40]	
<b>Panel C: Size/INV</b>					<b>Panel D: Size/OP</b>			
$H$	Constant	$\frac{KS_{t+H}}{KS_t}$	$\bar{R}^2$	$\frac{RMSE}{RMSR}$	Constant	$\frac{KS_{t+H}}{KS_t}$	$\bar{R}^2$	$\frac{RMSE}{RMSR}$
4	0.92	0.61	0.39	0.19	0.60	0.70	0.78	0.12
Base	[0.20, 1.54]	[0.27, 0.96]	[0.03, 0.70]		[0.26, 0.94]	[0.54, 0.87]	[0.48, 0.89]	
Under Null	[-0.09, 1.87]	[-0.02, 0.02]	[-0.04, 0.16]		[0.17, 1.02]	[-0.01, 0.01]	[-0.04, 0.16]	
8	1.70	0.55	0.62	0.16	1.61	0.57	0.76	0.12
Base	[1.50, 1.90]	[0.37, 0.74]	[0.29, 0.81]		[1.46, 1.77]	[0.45, 0.71]	[0.42, 0.90]	
Under Null	[1.43, 1.97]	[-0.01, 0.01]	[-0.04, 0.17]		[1.45, 1.77]	[-0.00, 0.00]	[-0.04, 0.16]	
<b>Panel E: All Equities</b>					<b>Panel F: All Assets</b>			
$H$	Constant	$\frac{KS_{t+H}}{KS_t}$	$\bar{R}^2$	$\frac{RMSE}{RMSR}$	Constant	$\frac{KS_{t+H}}{KS_t}$	$\bar{R}^2$	$\frac{RMSE}{RMSR}$
4	0.74	0.68	0.58	0.17	0.39	0.83	0.78	0.25
Base	[0.45, 1.01]	[0.54, 0.83]	[0.28, 0.73]		[-0.91, 0.63]	[0.71, 1.21]	[0.28, 0.79]	
Under Null	[0.37, 1.10]	[-0.01, 0.01]	[-0.01, 0.05]		[-0.26, 0.11]	[-0.01, 0.01]	[-0.01, 0.03]	
8	1.65	0.57	0.74	0.14	1.34	0.63	0.44	0.41
Base	[1.56, 1.74]	[0.49, 0.66]	[0.51, 0.84]		[0.81, 1.72]	[0.63, 0.96]	[0.42, 0.84]	
Under Null	[1.55, 1.74]	[-0.00, 0.00]	[-0.01, 0.05]		[0.51, 0.75]	[-0.00, 0.00]	[-0.01, 0.03]	

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