Price Elasticity

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I am very glad that you compare your result in this question to the one in first-year econ, however the concept in that course is *slightly* different

By slightly, I meant the concept is different in the definition of Price elasticity of demand.

Before I answer your question, let's first write down the total revenue function (TR)

The revenue function is simply price times quantity (simple as that)

$$TR = PQ(P)$$

Then how do we know whether or not TR is increasing or decreasing in P? Remember in your first year calculus I, to show some function is increasing/decreasing, we use the first derivative test. Therefore, we used that and conclude that

$$\begin{aligned} TR \text{ is increasing in } P \text{ if } \frac{\partial TR}{\partial P} &> 0 \\ TR \text{ is decreasing in } P \text{ if } \frac{\partial TR}{\partial P} &< 0 \\ TR \text{ is constant in } P \text{ if } \frac{\partial TR}{\partial P} &= 0 \end{aligned}$$

(that's basically first-derivative test).

Now our task reduce to find the first derivative of TR

$$\begin{aligned} \frac{\partial TR}{\partial P} &= \frac{\partial}{\partial P}(PQ(P)) \\ &= Q(P) + PQ'(P) \end{aligned}$$

where the last line, I used the chain rule.

Now, we can see that

$$\frac{\partial TR}{\partial P} > 0 \text{ if and only if } Q(P) + PQ'(P) > 0$$

But

$$Q(P) + PQ'(P) > 0 \text{ implies } \frac{PQ'(P)}{Q(P)} > -1$$

Therefore, we see that TR is **increasing** in P if and only if $\frac{PQ'(P)}{Q(P)} > -1$

1 First-Year Econ

In the principle econ course, the price elasticity of demand is defined as

$$E_D = \left|\frac{PQ'(P)}{Q(P)}\right|$$

Note that it's defined in **absolute value**, thus when $\frac{PQ'(P)}{Q(P)} > -1$ (we know that it is when TR is **increasing** in P) , its absolute value $\left|\frac{PQ'(P)}{Q(P)}\right| < 1$ (for example, -0.5 is greater than -1, but its absolute value 0.5 is smaller than 1), thus we see that TR is increasing in P when $E_D = \left|\frac{PQ'(P)}{Q(P)}\right| < 1$.

Similarly, when $\frac{PQ'(P)}{Q(P)} = -1$ (we know that it is when $\frac{\partial TR}{\partial P} = 0$, and TR is constant), the abolute value $\left|\frac{PQ'(P)}{Q(P)}\right| = 1$, thus TR is constant when $E_D = \left|\frac{PQ'(P)}{Q(P)}\right| = 1$.

Similarly, when $\frac{PQ'(P)}{Q(P)} < -1$ (we know that it is when $\frac{\partial TR}{\partial P} < 0$, and TR is decreasing), the abolute value $\left|\frac{PQ'(P)}{Q(P)}\right| > 1$ (for example, -5 is smaller than -1, but its absolute value 5 is greater than 1), thus TR is decreasing in P when $E_D = \left|\frac{PQ'(P)}{Q(P)}\right| > 1$.

Thus this is basically what the chart is saying.

2 Intermediate Micro

Now back to our course. In this course, we define the price elasticity of demand as

$$E_D = \frac{PQ'(P)}{Q(P)}$$

Note the difference is that, there is NO absolute value. Thus, we have

$$\begin{aligned} TR \text{ is increasing in } P \text{ if } \frac{PQ'(P)}{Q(P)} &> -1 \\ TR \text{ is decreasing in } P \text{ if } \frac{PQ'(P)}{Q(P)} &= -1 \\ TR \text{ is constant in } P \text{ if } \frac{PQ'(P)}{Q(P)} &< -1 \end{aligned}$$

3 For this question

Thus since the question is asking you that if TR is increasing in P when price elasticity is greater than 1, then

$$\frac{PQ'(P)}{Q(P)} > 1$$

and $1 > -1$, implies $\frac{PQ'(P)}{Q(P)} > -1$, thus from our definition TR is increasing in P

So in this course, as long as the price elasticity is greater than -1, TR is increasng in P