Intermediate Microeconomics

Recitation #1

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Introduction

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Course Tips

- Check message from blackboard frequently. (Not emailed to everyone somtimes)
- Feel free to email me regarding short question. Long question is welcomed but appointment is preferred.
- Make sure to fully understand the problem sets
  - Help understand the materials
  - Help prepare the midterm and final
- Practice and practice
  - Questions from textbook recommanded

Question 1

Problem

Joe has the following reference over the coffee shops: for any two coffee shops A,B, he weakly prefers A to B if a small cup of coffee is at least cheap at A as at B, and also A is at least as close (to his house) as B. Thus if p_i, d_i are the price, distance for coffee shop i,

$$A \succeq B \iff p_A \leq p_B \text{ and } d_A \leq d_B$$

Show that his preference relation $\succeq$ may not be complete

Question 1

Let's quickly refresh our memory about the basics of Preference. We use the following symbols to denote an individual's preferences between any two alternatives:

$\succsim$	means weak peference
$\succ$	means **strick** peference
$\sim$	means indiference

- Preferences are rational if $\succsim$ satisies two properties:
  - complete: for any two choices A, B, either $A \succsim B$ or $B \succsim A$ or *both*
    - when doing a questionnaire choosing A, B, the answer "I don't know" is not allowed
  - transitive: if $A \succsim B$ and $B \succsim C$, then $A \succsim C$

Question 1

Solution

For $\succsim$ to be complete, it must be that for any pair A, B, either $A \succsim B$ or $B \succsim A$ or both. This does not hold if there are two coffee shops A, B such that $p_A < p_B$ and $d_A > d_B$.

- Why?

Question 1

Solution

For $\succsim$ to be complete, it must be that for any pair A, B, either $A \succsim B$ or $B \succsim A$ or both. This does not hold if there are two coffee shops A, B such that $p_A < p_B$ and $d_A > d_B$.

- Why?
- The first inequality implies that $B \succsim A$ does not hold, since B is more expensive; the second implies that $A \succsim B$ does not hold, since A is further away.

Question 2

Problem

Explain graphically why indifference curves cannot cross

Question 2

Let's formally define the indifference curve

Definition

Let $\succsim$ be a preference relation on a set X.The indifference curve $IC(x)$ is a set of all $y \in X$ for which $y \sim x$

- In the set notation,

$$IC(x) = \{y \in X | y \sim x\}$$

Question 2

Solution

Graph on the Board. Suppose by contradiction that the indifference curve can cross. Then by IC_2, $A \sim B$. *But according to* IC_1, $A \succ C$ *and* $C \succ B$. *By transitivity, this implies* $A \succ B$, *contradiction.*

Question 3

Problem (3a)

Argue that the utility unction $v(x_1, x_2) = 2\ln x_1 + \ln x_2$ *represents the same preference as* $u(x_1, x_2) = x_1^{\frac{2}{3}} x_2^{\frac{1}{3}}$.

Problem (3b)

Write the equation of an indifference curve which represents these preferences and goes through the point (1,1)

Problem (3c)

If his consumer has twice as many units of good 1 as good 2, then what's his marginal rate of substitution (MRS) between the goods?

Question 3

Let's briefly talk about utility function.

Definition

We say that the function $U : X \to \mathbb{R}$ represents the preference $\succsim$ if for all x and $y \in X$, $x \succsim y$ if and only iff $U(x) \geqslant U(y)$.If the function U represents the preference relation $\succsim$, we refer to it has a utility function or we say that $\succsim$ has a utility representation.

Question 3

Theorem

If U represents $\succsim$, then fo any **strictly increasing** *function $f : \mathbb{R} \rightarrow \mathbb{R}$, the function $V(x) = f(U(x))$ represents $\succsim$ as well*

- Proof?

Question 3

Proof.

$$
\begin{aligned}
x &\succsim y \\
&\Longleftrightarrow U(x) \geqslant U(y) \text{ (since U represents } \succsim \text{)} \\
&\Longleftrightarrow f(U(x)) \geqslant f(U(y)) \text{ (since f is strictly increasing)} \\
&\Longleftrightarrow V(x) \geqslant U(y)
\end{aligned}
$$

□

Question 3

Back to our problem

Problem (3a)

Argue that the utility unction $v(x_1, x_2) = 2\ln x_1 + \ln x_2$ *represents the same preference as* $u(x_1, x_2) = x_1^{\frac{2}{3}} x_2^{\frac{1}{3}}$.

Solution

Here $v = \ln(u^3)$. *This is a* **strictly increasing** *function of u (why?)and hence by the theorem, v represents the same preference.*

Question 3

Problem (3b)

Write the equation of an indifference curve which represents these preferences and goes through the point (1,1)

Solution

We want all points (x_1, x_2) *satisfying* $(x_1, x_2) \sim (1, 1)$: *since u represents the preferences, this requires*

$$\begin{aligned} u(x_1, x_2) &= u(1, 1) \\ \Leftrightarrow \quad & x_1^{\frac{2}{3}} x_2^{\frac{1}{3}} = 1 \end{aligned}$$

- Note you can also use v, in that case the equation is $2 \ln x_1 + \ln x_2 = 0$ (verify)
  - Graph?

Question 3

Problem (3b)

Write the equation of an indifference curve which represents these preferences and goes through the point (1,1)

Solution

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  - Graph?
  - plot $x_2 = \frac{1}{x_1^2}$

Question 3

For 3(c), let's review marginal rate of substitution (MRS)

- tells you the rate at which you would give up y to get a bit more x
  - Formula in this case:
$$MRS_{(x,y)} = \frac{MU_x}{MU_y}$$
  - Negative slope of the indifference curve
  - Diminishing MRS implies preferences are **convex**

Question 3

Problem (3c)

If his consumer has twice as many units of good 1 as good 2, then what's his marginal rate of substitution (MRS) between the goods?

Solution

$$\begin{aligned} MU_1 &= \frac{\partial u}{\partial x_1} = \frac{2}{3}(\frac{x_2}{x_1})^{\frac{1}{3}} \\ MU_2 &= \frac{\partial u}{\partial x_2} = \frac{1}{3}(\frac{x_1}{x_2})^{\frac{2}{3}} \\ So,\ MRS_{(x_1,x_2)} &= \frac{MU_1}{MU_2} = \frac{2x_2}{x_1} \\ &\Rightarrow MRS_{(x_1,x_2)} = 1 \ when \ x_1 = 2x_2 \end{aligned}$$

- Mush less tedious if using $v(x_1, x_2)$ instead of u.

Question 4

Problem

John always eats his ballpark hotdog in a special way: he uses a footlong hotdog with precisely half a bun, 1 ounce of mustard, and 2 ounces of pickle relish. His utility depends only on these four items, and any extra amount of a single item (without the others) is worthless to him. What form does his utility function have?

- Solution?

Question 4

Problem

John always eats his ballpark hotdog in a special way: he uses a footlong hotdog with precisely half a bun, 1 ounce of mustard, and 2 ounces of pickle relish. His utility depends only on these four items, and any extra amount of a single item (without the others) is worthless to him. What form does his utility function have?

- Solution?
- $u(h, b, m, r) = \min(h, 2b, m, 0.5r)$

Question 4

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- Solution?
- $u(h, b, m, r) = \min(h, 2b, m, 0.5r)$
- $(1, 0.5, 1, 2) \sim (1, 0.5, 800, 2)$

Convexity

Definitions

Preferences $\succsim$ are **convex** if $x \succsim y$ implies $\alpha x + (1-\alpha)y \succsim y$, $\forall \alpha \in (0,1)$.

Definitions

Preferences $\succsim$ are **strictly convex** if $a \succsim y$, $b \succsim y$ and $a \neq b$implies $\alpha a + (1-\alpha)b \succ y$, $\forall \alpha \in (0,1)$.

Convexity

- In the footnote in chapter 3 of textbook, there is a fancy way to check if the preference is convex. Denote f be the utility function, then the represented preference is strictly convex if

$$f_2^2 f_{11} - 2 f_1 f_2 f_{12} + f_1^2 f_{22} < 0$$

- So if the cross derivative is zero (i.e $f_{12} = 0$), then the preference is strictly convex if $f_{11} < 0$ and $f_{22} < 0$
- Examples
  - (Cobb-Douglas) $u(x, y) = \ln x + \ln y$
  - (Perfect Substitution) $u(x, y) = x + y$
  - (CES) $u(x, y) = \frac{x^\delta + y^\delta}{\delta}$
- You can always use diminishing MRS criterion to check if the preferences are convex

CES Utility Function

$u(x, y) = \frac{x^\delta + y^\delta}{\delta}$

- If $\delta = 1$, $u(x, y) = x + y$, (perfect substitution)
- If $\delta = 0$, $u(x, y) = \ln x + \ln y$, (Cobb-Douglas). Derivation using L'Hôpital's rule
- If $\delta = -\infty$, $u(x, y) = \min(x, y)$ perfect complement
- Derivation beyond the scope of this course