Intermediate Microeconomics

Recitation #2

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Announcement

Recitation slides can be found at my NYU website

- Address: **files.nyu.edu/sm4529/public/Econ-10.html**
- Solution to the exercises in the slides not included
- Annoucement regarding problem set, course information will be posted in addition to Andrea's
- My office hours is TBA, but appointment is always welcomed
- Feedback is warmly welcomed

Problem Set #1

Problem set #1 has been posted in the NYU class

- Due sharply in the next recitation on **September 24**
- Study in group is encouraged
- Write down your answer individually
- Office hours for Problem Set #1: Friday, 2pm-4pm at my office 819

Budget Set

Definitions

A budget set is a set of bundles that can be represented as

$$B(p, w) = \{x \in X | \ px \leq w\}$$

Example

Consider two-commodity space $\mathbb{R}^2$
The bundle $x = (x_1, x_2)$: x_1 is apple, x_2 is orange
The price $p = (p_1, p_2)$: p_1 is price of apple, p_2 is price of orange
w is the wealth
Then the budget line is

$$B(p, w) = \{x \in X | \ p_1 x_1 + p_2 x_2 \leq w\}$$

Utility Maximization (Consumer's Problem)

We can represent the consumer's problem as follows, using the notation for budget set

$$\max_{x \in B(p,w)} \quad u(x)$$

Example

Consider two-commodity space $\mathbb{R}^2$
The bundle $x = (x_1, x_2)$: x_1 is apple, x_2 is orange
The price $p = (p_1, p_2)$: p_1 is price of apple, p_1 is price of orange
w is the wealth
$u(x) = u(x_1, x_2) = \alpha \ln x + \beta \ln y$ (Cobb-Douglas)

$$\max_{x} \ \alpha \ln x + \beta \ln y$$

$$\textit{subject to} \quad p_1 x_1 + p_2 x_2 \leq w$$

Note $p_1 x_1 + p_2 x_2 \leq w$ means $x \in B(p, w)$

Monotonicity

Definition

The preference relation $\succeq$ satisfies **monotonicity** at the bundle y if for all $x \in X$, we have
(i) if $x_k \geq y_k$ for all k, then $x \succeq y$ (ii) if $x_k > y_k$ for all k, then $x \succ y$
The preference relation $\succeq$ satisfies monotonicity if it satisfies monotonicity at every $y \in X$

- Example:
  - Cobb-Douglas: $u(x, y) = \alpha \ln x + \beta \ln y$
  - Perfect complement (Leontief utility function): $u(x, y) = min(x, y)$
  - Perfect substituion: $u(x, y) = x + y$

Walras's Law

Theorem

If the preferences are monotonic, then any solution x to the consumer problem $\max_{x \in B(p,w)} u(x)$ is located on its budget curve and thus $px = w$

- Note that solution to the consumer problem is called demand function, denote as $x(p, w)$

Example

Consider two-commodity space $\mathbb{R}^2$
The bundle $x = (x_1, x_2)$. The price $p = (p_1, p_2)$. w is the wealth.
$u(x) = u(x_1, x_2) = \alpha \ln x + \beta \ln y$
Since Cobb-Douglas represent monotonic preference, by walras's law, the optimal allocation x^* satisfies

$$p_1 x_1^* + p_2 x_2^* = w$$

Walras's Law

In this slide, I will prove the Walras's Law which is not required for this course, but if you are interested and have any question regarding the proof, please feel free to ask me

Proof.

[Proof of Walras's Law] Suppose not, then $px < w$. Consider $\mathbb{R}^K$ commodity space. There is an $\epsilon > 0$ such that $p(x_1 + \epsilon, ...x_K + \epsilon) < w$. By monotonicity, $(x_1 + \epsilon, ...x_K + \epsilon) \succ x$, thus contradicting the assumption that x is optimal in $B(p, w)$ □

Locally non-satiation

The following definition is rigorous, just for background, not required for this course

Definition

The preference relation $\succeq$ is said to be locally nonsatiated at x if for every $\epsilon > 0$, there exists a $y \in X$ such that $||y - x|| \leq \epsilon$ and $y \succ x$. A preference relation is said to be locally nonsatiated if it is locally nonsatiated at every $x \in X$

- Monotonicity implies non-satiation
- Indifference curves have an upward U-shape and an increase in either good always increases the utility
- Example: Cobb-Douglas
- Counterexample: Preference with "thick" indifference curve

Solving Consumer's Problem

- If preferences are convex and locally non-satiated, then the optimal solution (x, y) satisfies

$$\frac{MU_x}{MU_y} = \frac{p_x}{p_y}$$

$$p_x x + p_y y = w$$

- Two equations to solve two unknowns
- $MRS_{(x,y)} = \frac{MU_x}{MU_y}$, so the first equation can be also written as $MRS_{(x,y)} = \frac{p_x}{p_y}$

Solving Consumer's Problem: Lagrangian

If we want to maximize a function $u(x, y)$ subject to a constraint $g(x, y) \leq 0$, set up a lagrangian

$$\mathcal{L} = u(x, y) - \lambda g(x, y)$$

and then maximize with repect to x, y, μ. This gives you three first-order condition (assuming interior solution)

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial x} &= 0 \\ \frac{\partial \mathcal{L}}{\partial y} &= 0 \\ \frac{\partial \mathcal{L}}{\partial \lambda} &= 0 \end{aligned}$$

So three equations with three unknowns, we can solve for x, y

Question 1

Problem

Suppose you want to maximize the utility function $u(x, y) = xy + 2x$.

1. Calculate MU_x *and* MU_y

2. Explan how you know that you can ignore "corner solutions"

3. Graph an indifference curve with utility level 10

4. Find the optimal bundle, if prices are $1 for good x, $4 for good y, and your wealth is $20

5. Show graphically what this point looks like.

Partial Derivatives

Problem

Let $f(x,y) = ye^{(x^2+y^2)}$

Find f_x, f_y and f_{xy}, where $f_x = \frac{\partial f(x,y)}{\partial x}$, $f_y = \frac{\partial f(x,y)}{\partial y}$, $f_{xy} = \frac{\partial^2 f(x,y)}{\partial x \partial y}$

Problem

Let $f(x,y) = x\sqrt{y} + y$

Find f_x, f_y

Problem

Let $f(x,y) = \ln(x^2 + y^2 - 16)$

Find f_x, f_y, f_{xx}

Problem

Let $f(x,y) = \frac{y}{3x^2+4y^2}$

Find f_x, f_y

Solution (Solution 2.1)

$f_x = 2xye^{(x^2+y^2)}$, $f_y = (2y^2+1)e^{(x^2+y^2)}$, $f_{xy} = (4xy^2+2x)e^{(x^2+y^2)}$

Solution (Solution 2.2)

$f_x = \sqrt{y}$, $f_y = \frac{x}{2\sqrt{y}} + 1$

Solution (Solution 2.3)

$f_x = \frac{2x}{x^2+y^2-16}$, $f_y = \frac{2y}{x^2+y^2-16}$, $f_{xx} = \frac{-2x^2+2y^2-32}{(x^2+y^2-16)^2}$

Solution (Solution 2.4)

$f_x = \frac{-6xy}{(3x^2+4y^2)^2}$, $f_y = \frac{3x^2-4y^2}{(3x^2+4y^2)^2}$