Recitation #4

Sai Ma[1]

1 Annoucement

- Problem set #2 is sharply due on Oct. 7th, Monday **in Class**
- Office hour for PS2 is Friday, Oct.4th, in my office
- Grades for PS1 is posted in NYU class, please check the discrepany
- You can also pick up your graded PS1 from me, please let me know.

2 Question

We are considering a generalized Cobb-Douglas utility function $U(x, y) = \alpha \ln x + \beta \ln y$, where $\alpha + \beta = 1$. Denote the price as p_x and p_y and denote the wealth as w

Problem 1 *If $p_x = p_y = 1$, and $w = 10$, find the optimal bundle and graph your result*

Problem 2 *Suppose now p_x increases to 2 while p_y and w unchanged, the optimal bundle and sketch your result in the same graph as in problem 1.*

Problem 3 *Calculate and graph the Compensating Variation(CV)*

Problem 4 *Calculate the change in consumption of good x due to the Substitution Effect (SE) and the change in consuption of good x due to Income Effect (IE). Indicate them in your graph*

Problem 5 *Do the same as problem (4) for good y*

Problem 6 *If $p_x = p_y = 1$, set up the expenditure minimization problem and solve for the conpensated (Hicksian) demand function and Expenditure function. Can you say something about the relation between your result and that for utility maximization problem?*

Solution 7 (Solution to Problem 6) *For the Expenditure Minimization problem, we have*

$$\min_{x,y} \; p_x x + p_y y$$

$$s.t \; \alpha \ln x + \beta \ln y \geq u$$

[1] sai.ma@nyu.edu

To solve it, we set up the Lagranging, we have (imposing $p_x = p_y = 1$)

$$\mathcal{L} = p_x x + p_y y + \lambda(u - \alpha \ln x - \beta \ln y)$$

The F.O.C gives

$$1 = \frac{\lambda\alpha}{x} \quad (1)$$
$$1 = \frac{\lambda\beta}{y} \quad (2)$$
$$\alpha \ln x + \beta \ln y = u \quad (3)$$

In which (1) and (2) give us

$$x = \lambda\alpha \Rightarrow \ln x = \ln \lambda + \ln \alpha \quad (4)$$
$$x = \lambda\beta \Rightarrow \ln x = \ln \lambda + \ln \beta \quad (5)$$

Then plug (4) and (5) into (3), we can solve for λ

$$\begin{aligned} &\alpha (\ln \lambda + \ln \alpha) + \beta (\ln \lambda + \ln \beta) = u \\ &\Rightarrow (\alpha + \beta) \ln \lambda = u - \alpha \ln \alpha - \beta \ln \beta \\ &\Rightarrow \ln \lambda = u - \alpha \ln \alpha - \beta \ln \beta \quad (6) \end{aligned}$$

where the (6) uses the assuption that $\alpha + \beta = 1$
Now plug (6) into (4) and (5) to solve for Hicksian demand x and y

$$\begin{aligned} \ln x &= \ln \lambda + \ln \alpha \\ &= u - \alpha \ln \alpha - \beta \ln \beta + \ln \alpha \\ &= u + (1 - \alpha) \ln \alpha - \beta \ln \beta \\ &= u + \beta \ln \alpha - \beta \ln \beta \\ &= u + \beta \ln \frac{\alpha}{\beta} \quad (7) \end{aligned}$$

where in the deriation I uses the fact that $\alpha + \beta = 1$ implies $(1 - \alpha) = \beta$
Similarly, we have

$$\begin{aligned} \ln y &= \ln \lambda + \ln \beta \\ &= u - \alpha \ln \alpha - \beta \ln \beta + \ln \beta \\ &= u + (1 - \beta) \ln \beta - \alpha \ln \alpha \\ &= u + \alpha \ln \beta - \alpha \ln \alpha \\ &= u + \alpha \ln \frac{\beta}{\alpha} \quad (8) \end{aligned}$$

Therefore (7) gives

$$\ln x = u + \beta \ln \frac{\alpha}{\beta}$$

And (8) gives

$$\ln y = u + \alpha \ln \frac{\beta}{\alpha}$$

Therefore, the Hicksian demand function is

$$x^H = e^{(u+\beta \ln \frac{\alpha}{\beta})} \quad (9)$$

$$y^H = e^{(u+\alpha \ln \frac{\beta}{\alpha})} \quad (10)$$

Therefore, by (9) and (10), we have the expenditure function

$$\begin{aligned} E(p_x, p_y, u) &= p_x x^H + p_y y^H \\ &= x^H + y^H \\ &= e^{(u+\beta \ln \frac{\alpha}{\beta})} + e^{(u+\alpha \ln \frac{\beta}{\alpha})} \end{aligned}$$

where I uses the assumption that $p_x = p_y = 1$