Bonus Materials
Recitation 7

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1 Expected Utility

During the recitation, I chatted a bit about a gamble involving NCAA tournament.

I think MSU has a very good chance of winning the title and my roommate, Mr. A, asked me if I would like to have a bet. In particular, he will give me 100 bucks if MSU wins and nothing happens if MSU loses. In addition, my another roommate, Mr. B, acting as intermediary, that collects a participation cost for this gamble. That is, I can only play this gamble with Mr.A after paying this fixed cost of \$10

I told him that my decision of playing or not playing depends on my subjective probability of MSU actually wins the tournament, and also depends on my utility function.

Intuitively, if I think the probability that MSU is going to win it all is one, I would definitely accept this gamble. On the other hand, if I think MSU has zero probability of winning the title, I will for sure walk away.

How do we model this story and if I accept this gamble, can you infer what my subjective belief in MSU's chance of winning the title is ?

Suppose I am risk neutral, having a utility function

$$u(W) = W$$

Since $u'' = 0$, I am risk neutral.

The state of the world in this case

$$s \in \mathcal{S} = \{win, lose\}$$

The associated probability is

$$\pi(s) = \begin{cases} p & for\ s = win \\ 1-p & for\ s = lose \end{cases}$$

The gamble is

$$W = \begin{cases} \$100 & \text{with probability } p \\ 0 & \text{with probability } 1-p \end{cases}$$

Now, before taking the gamble, even though I am confident that MSU has a good chance of winning, but the beauty of basketball is that it is full of uncertainty. Therefore, to make my decision, I would like to know what is my **expected utility** from taking this gamble, and use it to compare the cost from taking this gamle, which is \$10

My expected utility from taking this gamble is simply

$$\begin{aligned} E(U(w)) &= p(100) + (1-p)0 \\ &= 100p \end{aligned}$$

Thus I will accept this gamble if and only if

$$E(U(w)) \geq 10$$

This implies that

$$100p \geq 10 \implies p \geq 10\%$$

Thus I will accept this bet if I am confident that MSU has **at least** 10% probability of winning the title.

This is more like a Micro-type question for expected utility, but I hope that you can understand the expected utility and uncertainty (state of world, associated probability, etc) better

2 Trick on Solving Investment Problem using Recursive Form

In the recitation, I introduced a power trick for deriving the F.O.C w.r.t capital.

In particular, consider question 2 in homework 5

The recursive form of firm's problem $[FP]$ is

$$\begin{aligned} V_t &= \max_{k_{t+1}\geq 0} \left\{ (1-\tau_t)(z_t f(k_t) - p_t(k_{t+1} - (1-\delta)k_t)) + \frac{1}{1+r} V_{t+1} \right\} \\ & \quad k_0 \text{ is given} \end{aligned}$$

In the lecture note, professor derived the F.O.C using the substitution method. But it involves a bit algebra.

The key element in this trick is finding the **state variable**

State variable is a variable that firm has decided in the past but will also affect the investment decision today.

In this example, at time t, k_{t+1} is the choice variable (the one firm chooses at time t), and from looking at the time t maximization problem (just focus on t)

$$\max_{k_{t+1}} \pi_t = (1-\tau_t)(z_t f(k_t) - p_t(k_{t+1} - (1-\delta)k_t))$$

This problem depends on k_t and we know that k_t is already decided at time $t-1$. Therefore, k_t IS the state variable at time t

There is no other state variable, τ_t and p_t are determined by government and market respectively.

Therefore, the only state variable at time t is k_t. (and state variable at time $t+1$ is k_{t+1})
Now, we can write the value function using state variable

$$V_t\left(k_t\right) = \max_{k_{t+1}\geq 0}\left\{\left(1-\tau_t\right)\left(z_t f\left(k_t\right) - p_t\left(k_{t+1} - \left(1-\delta\right)k_t\right)\right) + \frac{1}{1+r}V_{t+1}\left(k_{t+1}\right)\right\}$$

To find the F.O.C, following the three steps below
Firstly, taking the F.O.C with repsect to k_{t+1}

$$\left[k_{t+1}\right] : \left(1-\tau_t\right)\left(-p_t\right) + \frac{1}{1+r}V'_{t+1}\left(k_{t+1}\right) = 0 \tag{1}$$

Secondly, taking the F.O.C with repect to (state variable) k_t

$$\left[k_t\right] : V'_t\left(k_t\right) = \left(1-\tau_t\right)\left(z_t f'\left(k_t\right) + p_t\left(1-\delta\right)\right)$$

Note that this implies that (change t to $t+1$)

$$V'_{t+1}\left(k_{t+1}\right) = \left(1-\tau_{t+1}\right)\left(z_{t+1} f'\left(k_{t+1}\right) + p_{t+1}\left(1-\delta\right)\right) \tag{2}$$

Thirdly, plug (2) into (1)

$$\left(1-\tau_t\right)\left(-p_t\right) + \frac{1}{1+r}\left(\left(1-\tau_{t+1}\right)\left(z_{t+1} f'\left(k_{t+1}\right) + p_{t+1}\left(1-\delta\right)\right)\right) = 0$$

You are done!!
Rest of the job is simply re-arranging terms, and we have the Jorgenson's investment choice

$$\left(1-\tau_t\right)p_t = \frac{\left(1-\tau_{t+1}\right)\left(z_{t+1} f'\left(k_{t+1}\right) + p_{t+1}\left(1-\delta\right)\right)}{1+r}$$

This method dramatically reduces the algebra, but the key difficulty is to find the correct state variable. It's your judgement call to decide what method to be applied in the exam/homework.