Math and Stats Review

Sai Ma*

April 24, 2014

This file is the math and stats review for the concepts needed for the search theory.

Random Variable

When I covered uncertainty, we used the discrete random variable. For example, X is a random variable defined as

$$X = \begin{cases} 0 & w.p\ 0.2 \\ 1 & w.p\ 0.3 \\ 2 & w.p\ 0.5 \end{cases}$$

X takes value of $0, 1, 2$, since it is discrete, it is called **discrete random variable**

However, if X takes continuous values, for example any values in interval $[0, 1]$, then it is called **continuous random variable**.

PDF, CDF and Expectations

For discrete random variable X we have above, the expectation is

$$\begin{aligned} \mathbb{E}(X) &= \sum_{X \in \{0,1,2\}} p(X) X \\ &= (0.2)\,0 + (0.3)\,1 + (0.5)\,2 \\ &= 1.3 \end{aligned}$$

As for the continuous random variable, we cannot express expectation as a summation, since it takes a continuum of numbers (e.g $[0, 1]$), therefore, we use **integral** instead (remember integral is just summation of continuum of numbers) Therefore, we have

$$\mathbb{E}(X) = \int x f_X(x)\, dx \tag{1}$$

where $f_X(x)$ is called **probability density function** (PDF).

The **cumulative distribution function** (CDF) $F_X(x)$ describes the probability that a random variable X with pdf $f_X(x)$ will be found to have a value less than or equal to x.

$$F_X(x) = \Pr(X \le x) = \int_{-\infty}^{x} f_X(t)\, dt$$

Suppose $F_X(x)$ is defined on the support $[x_{\min}, x_{\max}]$, three properties you need to remember

*Email: sai.ma@nyu.edu.

- $f_X(x)\,dx = dF_X(x)$
- $F_X(x_{\max}) = 1$ and $F_X(x_{\min}) = 0$
- $\int_a^b dF(x) = F(b) - F(a)$ (Fundamental Theorem of Calculus)

Therefore, using property 1, we can also express the expectation in terms of CDF.

$$\mathbb{E}(X) = \int x dF_X(x) \tag{2}$$

(2) is commonly used to express expectation in terms of CDF in the search theory.

Leibniz Integral Rule

Leibniz rule is commonly used when you derive the comparative statics regarding the reservation wage w^* in the search model. It is essentially a formula for the partial derivatives of the integral.

$$\frac{d}{dx}\left(\int_{a(x)}^{b(x)} f(t,x)\,dt\right) = f(b(x),x)\,b'(x) - f(a(x),x)\,a'(x) + \int_{a(x)}^{b(x)} f_x(t,x)\,dt$$

Example:

$$\begin{aligned} \frac{d}{dx}\left(\int_{2x}^{5x^2} t \ln x dt\right) &= \left(5x^2 \ln x\right)(10x) - (2x \ln x)(2) + \int_{2x}^{5x^2} \frac{t}{x} dt \\ &= 50x^3 \ln x - 4x \ln x + \int_{2x}^{5x^2} \frac{t}{x} dt \end{aligned}$$